UNIVERSITY OF MICHIGAN
ENGINEERING RESEARCH INSTITUTE

ELECTRONIC DEFENSE GROUP TECHNICAL MEMORANDUM NO. 29

BY: R. R. McPherson

DATE: July 10, 1956

SUBJECT: Use of Hewlett-Packard Model 522B Electronic Counter as a "Spinning Disc" Random Number Selector

ABSTRACT

Manual methods are described for selecting uniformly distributed random numbers by means of an electronic counter. Their randomness is illustrated by a trial result. A method for converting uniformly distributed random numbers to arbitrarily distributed random numbers is illustrated.

I. INTRODUCTION

The "spinning disc" random number selector* is a device for choosing decimal digits at random. The name derives from a construction utilizing a flash lamp to stop the motion of a number-carrying disc so that an operator can read the digit exposed through a window in a cover, behind which the disc is rotated by a fast motor.

Other forms of spinning-disc random number selector may be more convenient. In particular, an indication of the selected number which can be held indefinitely may be desired. Simultaneous selection of several independent digits may also be useful. These and other characteristics may be obtained through use of the Hewlett-Packard Model 522B electronic counter, in manual operation.

* See references.

II. METHODS FOR OPERATION OF THE COUNTER

Two methods may be used for manual selection of random numbers which are uniformly distributed over the set of four-digit decimal numbers. The first method uses only the counter and the manual gate function. The second method uses the counter and an adapter-box for external operation of the start-stop trigger circuits of the time interval function. Basically, the counter circuits are manually started in counting the interval 100 kc/s standard frequency. In this condition "the disc is spinning". Then the counter is manually stopped and the count read. The right-hand four digits constitute a selection from a distribution uniform in density over the 10,000 numbers 0000 to 9999 inclusive.

A. The adjustments for the HP-522B using the first method are:

(1)	Function selector:	Manual gate
(2)	Time unit switch:	100 kc/s standard frequency counted
(3)	Display time:	CCW (Minimum).

With the Manual Gate Switch in the OPEN position, the count is revolving. When the Manual Gate Switch is operated to the closed position, the count may be read. Only the right-hand four digits may be recorded as a random number.

B. The adjustments for the HP-522 B using the second method are:

(1)	Trigger level:	Start: + 20 v Stop: + 20 v
(2)	Trigger slope:	Start: - Stop: +
(3)	Trigger input switch:	Common
(4)	Time unit switch:	100 kc/s standard frequency counted
(5)	Function selector:	Time interval
(6)	Display time:	CCW (minimum)
(7)	Phototube voltage:	connected to input connector of adapter switch-box

(8) Trigger input (start or stop): Connected to output connector of adapter switch-box.

With the switch on the Adapter Switch-Box operated to either side and returned to center position, the count is revolving. When the switch on the Adapter Switch-Box is subsequently operated to either side position, the count is displayed. Only the right-hand four digits may be recorded as a random number.

III. CONSTRUCTION OF THE ADAPTER SWITCH-BOX

An adapter switch-box, which permits remote operation of the selection of a random number, can incorporate a lever-type switch specifically designed for large numbers of repeated throws, thus saving the Manual Gate toggle switch for its intended function of occasional use. Such an adapter which forms a manual trigger voltage from the phototube voltage is diagrammed in Figures 1, 2, and 3. A 0.5 μf 200 v metallized paper capacitor is employed to eliminate the effects of contact bounce in the switch, even when it is abruptly operated. In addition, the polarity of start-stop functions is arranged to employ the smooth contact closure available when operating the lever switch from its center position to the side position for stopping the count. Because of the automatic reset function of the interval measuring circuits in the counter, unilateral action in the stopping of the count is essential for the count to be recorded as a random number. This fact requires a trigger pulse free of the effects to contact bounce at the time of stopping of the count. Otherwise, a small count due to the length of a bounce will be the count which is registered.

IV. RANDOMNESS OF THE NUMBERS SELECTED

The numbers selected in either of the two manual methods described in Section II are random in the usual sense simply as a result of indefiniteness in human timing with respect to intervals of length 0.01 sec. It is assumed that the operator is not permitted to time his actuation of the switch lever from the fifth counter register whose digits appear at a rate of 10 per second. This result may be accomplished either through covering the fifth register or simply ignoring the counter in operation. Figure 4 shows the results obtained in one sequence of 250 four-digit numbers, using the latter method and the adapter switch-box. A total of 1000 digits is recorded.

The probability functions shown in Figure 4 have been defined on the set of ten digits. That is, the four digits of a single reading have been lumped onto a composite space obtained through using the fact that the method of selection makes the four digits statistically independent of one another. This procedure is simply a second interpretation of the meaning of a uniform distribution over the 10,000 four-digit numbers. For example, by a similar extension, a binary distribution over the digits 0, 1 can be obtained by recording a digit as 0 if it is even, as 1 if it is odd.

Table 1 summarizes a comparison of the trial distribution function of Figure 4 with the theoretical uniform distribution in respect to a few parameters. The point of view is that no physical process of uniform distribution exists; it is only that the probability model for comparison is that of the uniform distribution. Satisfactory agreement is evident in Table 1.

i	x_i	f_i	Fig. 4 Trial Distribution	Uniform Distribution
1	0	83	$n = \sum_{i=1}^{10} fi = 100$	Each fi = 100
2	1	94		
3	2	87	Median = 5	Any x, where $4 \le x < 5$
4	3	105	Mode = 5	Undefined
5	4	98	Mean $= 1/n \sum_{i=1}^{10} x_i f_i$	
6	5	118	= 4.683	4.500
7	6	108	Second Moment $= 1/n \sum_{i=1}^{10} x_i^2 f_i$	
8	7	101	= 29.881	28.50
9	8	91	Mean-squared = 21.930489	20.25
10	9	115	Variance = 7.950511	8.25
		1000	Std. Dev. = 2.8197	2.8723

TABLE 1 Comparison of Parameters of a Trial Distribution and a Theoretical Distribution Function

2262A-S3-57 MB 7-2-56

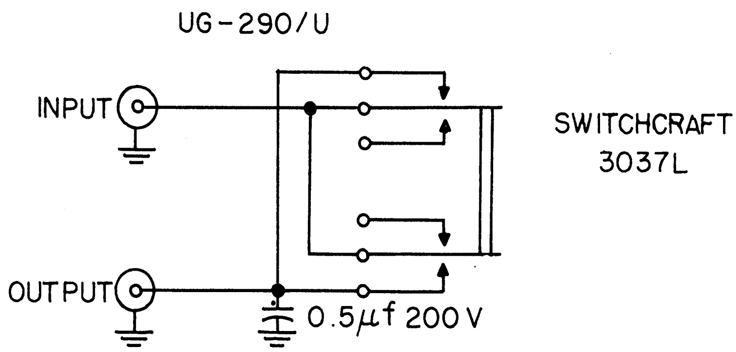

FIGURE 1 ADAPTER SWITCH-BOX

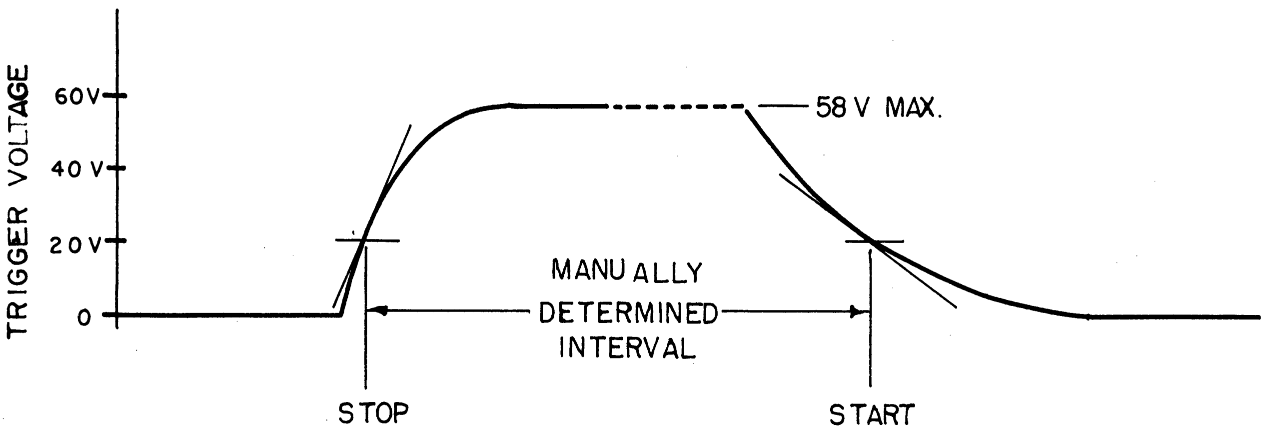

FIGURE 2 MANUAL TRIGGER FORMED BY ADAPTER SWITCH-BOX

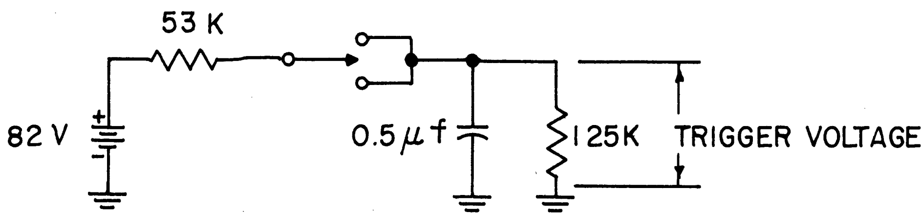

FIGURE 3 EQUIVALENT CIRCUIT OF ADAPTER SWITCH-BOX, PHOTOTUBE VOLTAGE SOURCE AND TRIGGER INPUT LOAD

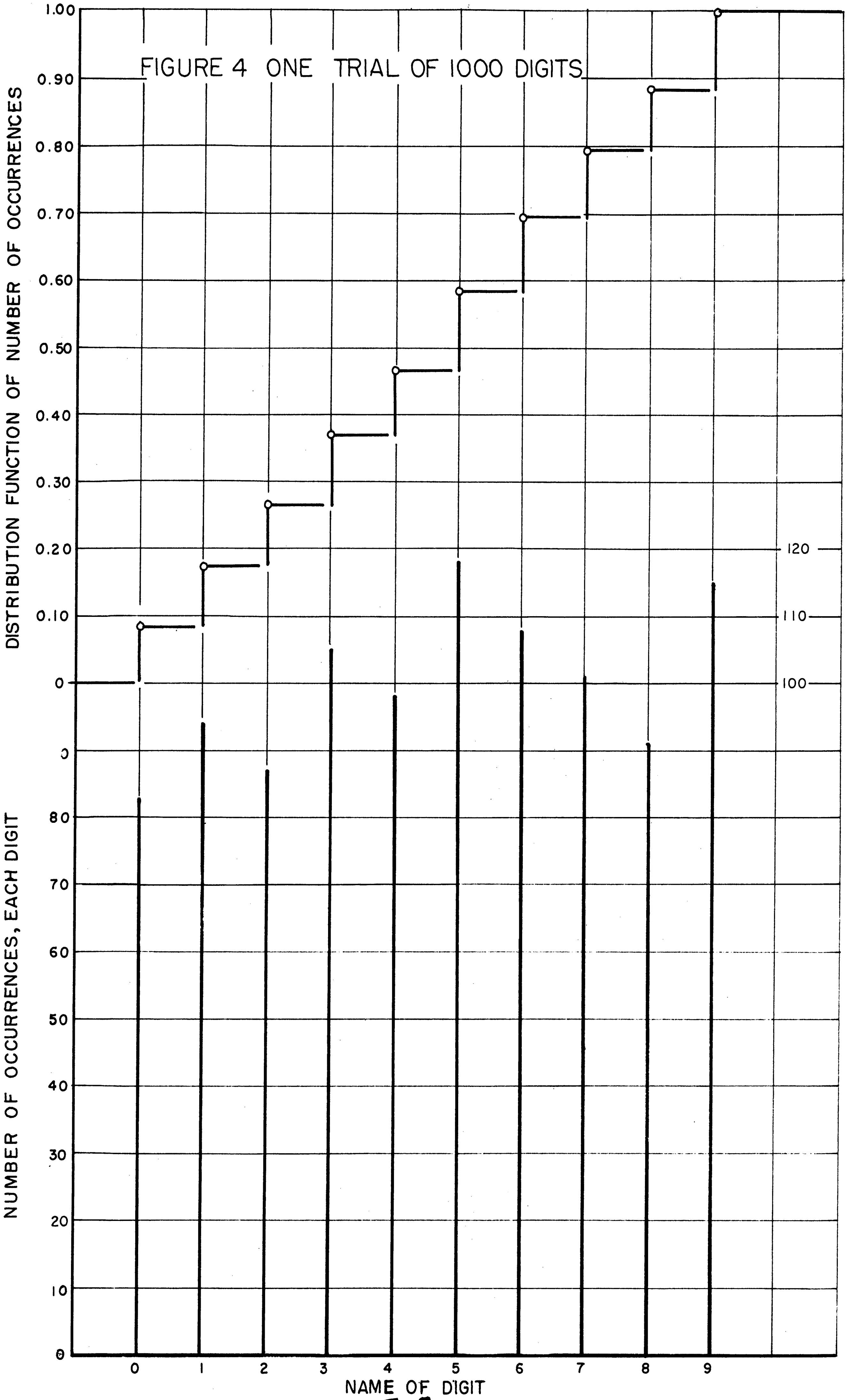
FIGURE 4 ONE TRIAL OF 1000 DIGITS
DISTRIBUTION FUNCTION OF NUMBER OF OCCURRENCES
NUMBER OF OCCURRENCES, EACH DIGIT
NAME OF DIGIT
1.00
0.90
0.80
0.70
0.60
0.50
0.40
0.30
0.20
0.10
0
80
70
60
50
40
30
20
10
0
120
110
100
0
1
2
3
4
5
6
7
8
9

A χ^2 test of the f_i, i=1, 2, ... 10, of Table 1 may be made against this uniform distribution, with each p_i = 0.10 and n = 1000. Then[1]

$$(1) \qquad \chi^2 = \sum_{i=1}^{r} \frac{(f_i - np_i)^2}{np_i} = \sum_{i=1}^{10} \frac{(f_i - 100)^2}{100} = 12.18,$$

$$(2) \qquad E(\chi^2) = r-1 = 9,$$

$$(3) \qquad D^2(\chi^2) = 2(r-1) + \frac{1}{n}\left[\sum_{i=1}^{r} \frac{1}{pi} - r^2 - 2r + 2\right]$$

$$= 2(9) + \frac{1}{1000}\left[\sum_{i=1}^{10} 10 - 100 - 20 + 2\right]$$

$$= 18 - \frac{18}{1000} = 17.982, \text{ and}$$

$$(4) \qquad D(\chi^2) = \sqrt{D^2(\chi^2)} = \sqrt{17.982} = 4.2405$$

The observed χ^2 = 12.18 for the f_i of Table 1 is within plus one standard deviation of the mean value 9, since $9 < 12.18 < 13.2405$. Reference to a table of the theoretical limiting χ^2 distribution, with 9 degrees of freedom, gives a theoretical χ^2 value of 16.919 at the 5% level of significance, 12.242 at the 20% level of significance, and 10.656 at the 30% level of significance. Thus the observer χ^2 = 12.18 would on the average be exceeded in only about one out of five trials of the kind leading to Figure 4 and Table 1. The observed χ^2 is not significant at the usual level of 5%.

Perhaps the best model for comparison with the f_i of Table 1 is the multinomial distribution, with each probability p_j of success equal to 0.1, j = 1,2 .... 9, 10, and n = 1000. Then the following expected value formulas[1] apply:

1. H. Cramer, Mathematical Methods of Statistics, Princeton University Press, 1951, Cf. p. 417, and p. 318.

(5) $m_j = E(x_j) = np_j$, $j = 1,2, \dots 10.$

(6) $\lambda_{jk} = E\left[(x_j - np_j)(x_k - np_k)\right] = -np_j pk, \quad k \neq j$
$= np_j(1-p_j), \quad k = j$ } $j,k = 1,2 \dots 10.$

Evaluating these formulas, we have

(7) $m_j = np_j = (1000)(0.1) = 100$ $j = 1,2, \dots 10,$

(8) $\lambda_{jk} = -np_j p_k = -(1000)(0.1)(0.1) = -10$, $j,k = 1,2, \dots 10, \; k \neq j$

and

(9) $\lambda_{jk} = np_j(1-p_j) = (1000)(0.1)(0.9) = 90,$ $j = 1,2 \dots 10, \; k = j.$

Table 1 provides a comparison of f_i, i=1, ... 10, with the mean m_j = 100, j = 1, ... 10, and Table 2 provides a comparison of $(f_i - m)^2$, i = 1, ... 10.

i	x_i	f_i-m	$(f_i-m)^2$
1	0	-17	289
2	1	-6	36
3	2	-13	169
4	3	+5	25
5	4	-2	4
6	5	+18	324
7	6	+8	64
8	7	+1	1
9	8	-9	81
10	9	+15	225
Std. Dev. :		9.487	
Variance:			λ_{jj}= 90

TABLE 2. Comparison of Table 1 f_i with Multinomial Variance and Standard Deviation

with the variance $\lambda_{jj} = 90$, j = 1, ... 10, and of $|f_i - m|$, i = 1, ... 10, with the standard deviation $\sqrt{\lambda_{jj}} = 9.487$, j = 1, 2 ... 10. Table 3 displays the coproducts $(f_j-m)(f_k-m)$, j,k = 1,2, ... 10, $k \neq j$, each to be compared with the expected value $\lambda_{jk} = -10$, $k \neq j$, j,k = 1,2 ... 10. In summary, several simple tests based on probability models give acceptable demonstration of randomness in the data of Figure 4 taken by manual operation of the HP-522B as a random number selector.

k \ j	1	2	3	4	5	6	7	8	9	10
1	--									
2	+102	--								
3	+221	+78	--							
4	-85	-30	-65	--						
5	+34	+12	+26	-10	--					
6	-306	-108	-234	+90	-36	--				
7	-136	-48	-104	+40	-16	+144	--			
8	-17	-6	-13	+5	-2	+18	+8	--		
9	+153	+54	+117	-45	+18	-162	-72	-9	--	
10	-255	-90	-195	+75	-30	+270	+120	+15	-135	--

TABLE 3. Tabulation of Coproducts

V. DERIVATION OF NUMBERS DISTRIBUTED NONUNIFORMLY

From numbers distributed uniformly, one can derive numbers distributed arbitrarily through inverse use of the desired arbitrary distribution function of Figure 4, continuous on the right, is the desired arbitrary distribution function.* Then a selection of a four-digit random number, as described in Section II, is employed graphically as follows. Suppose 4505 were the selection. Entering at 0.4505 as ordinate, the intersection with the step-function curve continuous on the right is observed to be the digit 4. Then 4 becomes the sample of a random number distributed according to Figure 4. If the desired distribution function $\Phi(t)$ were the normal distribution function with mean equal to zero and variance equal to one, where

(10) $$\Phi(t) = \frac{1}{\sqrt{2\pi}} \int_{-\infty}^{t} \exp\left(-\frac{x^2}{2}\right) dx ,$$

then a table[1] of the function $\Phi(t)$ entered inversely at 0.4505 gives $t = -0.1244$, interpolated linearly between $\Phi(-0.13) = 0.4483$ and $\Phi(-0.12) = 0.4522$. Then $t = -0.1244$ becomes the corresponding sample of a random number distributed so as to be normal (0, 1). Similarly, if Pearson's Type III function, skewness 1.1, were the desired distribution,[1] then $t = -0.2939$ becomes the corresponding sample number. In these last two cases, the possible distribution covers the entire range of real numbers, not merely the decimal digits. Also, full use of the referenced tables would require a six-digit uniformly distributed random number. Two selections by either method of Section II would then suffice, say by joining the last three digits of the two selections. As a last example, five of these digits may be used inversely with a five-place table of common logarithms to derive random numbers x between 1 and 10 distributed with density proportional to $1/x$. The distribution function for this case is defined by

(11) $\Pr(x \leq t) = F(t)$, and

* The step rises are associated with the absicissae at which they are drawn, and the curve is continuous two-dimensional excepting only for the ten points at the bottoms of the rises and right-ends of the runs. These points are deleted for unique inversion.

1. H. C. Carver, Mathematical Statistical Tables, Edwards Brothers, Ann Arbor, Michigan, 1950, Cf. p. 20, Skewness = 0 and p. 21 Skewness = 1.1.

$$(12) \qquad F(t) = 0, \qquad t < 1;$$
$$= \log_{10} x, \qquad 1 \leq x \leq 10;$$
$$= 1, \qquad 10 < x.$$

VI. CONCLUSIONS

Either of two manually operated methods may be employed to use the Hewlett-Packard 522B Electronic Counter as a "Spinning-Dics" Random Number Selector. The methods as described may be regarded as producing numbers uniformly distributed over the set of four-digit decimal numbers. Graphed or tabular forms of arbitrary distribution functions may be used to derive random numbers distributed in aribtrary fashion from the numbers produced by manual operation of the electronic counter.

REFERENCES

1. R. W. Walker, "An Electronic Random Selector," Jour. Brit. I.R.E., 14:262-268, June, 1954.

2. M. G. Kendall and B. Babington Smith, "Second Paper on Random Sampling Numbers," Supp. to Jour. Roy. Stat. Soc., 6:51-61, 1939.

DISTRIBUTION LIST

60 Copies	Transportation Officer, SCEL Evans Signal Laboratory Building No. 42 Belmar, New Jersey FOR - SCEL Accountable Officer Inspect at Destination File No. 22824-PH-54-91(1701)
1 Copy	H. W. Welch, Jr. Engineering Research Institute University of Michigan Ann Arbor, Michigan
1 Copy	J. A. Boyd Engineering Research Institute University of Michigan Ann Arbor, Michigan
17 Copies	Electronic Defense Group Project File University of Michigan Ann Arbor, Michigan
1 Copy	Engineering Research Institute Project File University of Michigan Ann Arbor, Michigan
1 Copy	Document Room Willow Run Laboratories University of Michigan Ypsilanti, Michigan